Scholarship Research Report
Fall 2003

Investigations into the Application of Cumulant Functions in Operations Research and Stochastic Modeling

by
Martijn Kolloffel

TABLE OF CONTENS

This report is a chronological documentation of the research progress made by Martijn Kolloffel throughout the Fall Semester 2003. The research focuses on the use of cumulant functions in queueing theory and stochastic modeling. This report is a result of a DOD research grant proposed by Dr. Timothy. I. Matis, with the purpose to engage undergraduate students from New Mexico State University in research in stochastic models. My progress was monitored, evaluated, and documented through the participation in the undergraduate research course IE 400. The investigation into the application of cumulant functions in stochastic modeling is a continuation of research activities in the spring semester of 2003.

The usefulness of cumulant based analysis methods is researched by the formulation of a suitable model. The first goal in this semesters research endeavor is the definition of a model that is relevant to military applications. After the model is completely defined, we use a cumulant derivation procedure to find an approximation to the measure of interest. We then validate our solution by comparing it to a model simulation. After validation we strive to expand the scale of the initial model, which can show the mathematical tractability of this procedure for large-scale systems. We also intend to expand the model by using phase type distributions, which allows us to model most probability distributions.
During the summer of 2003 some ideas for formulating a model suitable for cumulant-based modeling had been generated in relation to a NASA research grant, that focused on the reliability of components that are used in space technology. We think that the stochastic analysis of component reliability in military applications can also be relevant and useful to the Department of Defense. Optimizing maintenance schedules of military equipment can prevent critical system failure and improve system availability.

The first papers in my literature research include the "Analysis of equipment availability under varying corrective maintenance models" by Cassady and Iyoob. [1], and a paper by M. Kijima: "Some results for repairable systems with general repair" [2]. Reading these papers resulted in thinking about availability or reliability of systems. The initial model we formulate is a gearbox in which the rate of failure is dependent on the state of the individual variables in the gearbox. These variables could be the temperature, pressure, or the number of particles in the box. We assumed that these variables of the gearbox are in a state of no,

light, moderate, or severe wear. We are not completely satisfied with our formulation and foresee some problems when applying the cumulative procedure to this model, which inspires me to step up the library research.

I.J. Rehmert generates my interest in shock models after reading his dissertation: "Time dependent availability analysis for the Quasi-Renewal process". Each shock arriving to the system causes component failure that is a function of the total accumulated damage from previous shocks. The components are monitored at discrete points in time, which can be modeled with a discrete aging process.
This leads us to investigate compartmentalized wear models involving crack propagation. We formulate a model in which cracks initiate discretely according to a Poisson distribution, and propagate according to a continues non-linear function. The damage at time t can be represented as an integral of an analytical function, the "crack growth expression". I tried viewing crack propagation as a deteriorating system, but once again encountered an expansion of the model that was unwanted at this point in time. In order for me to be able to construct an analytical and simulated model within the timeframe given I opted for a more simplistic model, which would stress the advantages of using cumulative derivation. This brought us right back at thinking about component reliability.

We formulate a model in which a single component operates in an environment of non-fatal repairable shocks, or impacts. These shocks represent system stresses or other components failing in the system. The shocks will cause repairable and cumulative damage to the component. The arrival of shocks is according to a Poisson distribution. The failure of the component is a result of both the un-repaired or current and the cumulative shocks at time t. We can write an intensity function corresponding to component failure that is a nonlinear function of the current and cumulative number of shocks. We can now use the cumulant derivation method to obtain an approximation to the probability that the component is operational at time t. An extended explanation of the model described above can be found in the appendix.

To great relief we finally formulate a model that seems to fit the scope of my research.

The next step is the creation of numerical results based on our analytical model. Mathematica software, and a program for the Cumulant derivation procedure, which Dr. T.I. Matis has developed, helps us solve our partial differential equations. We define several intensity functions and evaluate the availability of the component at time t, letting t increase discretely from 0 to 50. The results are imported in excel to make a graphical comparison with the simulated results possible.

We want to evaluate how well the approximation of the availability of the component is. A simulation in ProModel simulation software is built to compare the approximation to. The modeled system seems simple enough to simulate. In thinking about the exact timing of every event in the simulation however, I realized that discrete event simulation software is not always specifically designed to simulate analytical models. Debugging the simulation with the trace-function is necessary to ensure that the simulation is operating exactly as stated in the model formulation. To obtain a 5% confidence interval on our simulation data, we calculate that around 40.000 replications are needed. The times of failure of the component are written to a text-file, that is imported in excel. [Appendix]

In looking at the resulting graphs for the expected time of failure of the component we can conclude that the cumulative derivation method is fairly accurate. We also see that it's accuracy increases as the truncation level increases, which seems to be a logical observation. The mathematical tractability as the system size increases is this methods greatest strength. This semester is coming to a rapid close, and I regret not being able to explore the advantages this method has when we increase the size of the system. The goal to expand the model to phase-type distributions has also not been realized. I realize that knowledge is the most abundant resource in nature, and that there is still a lot of work to be done. I would like to thank everybody who has made it possible for me to be exposed to the practice of doing research and the field of stochastic modeling in particular. My interest is to continue as a graduate student in the field of Industrial Engineering, specifically in the areas of operations research and simulation modeling. Special thanks to Dr.T.I. Matis who challenged, inspired, and supported me throughout this entire experience.

Library Research

Cassady, C.R., Iyoob, I.M., "Analysis of Equipment Availability under Varying Corrective Maintenance Models". Work Review (2002)

In this paper, the authors present availability measures of repairable equipment. Different corrective maintenance models are compared and the effects of varying parameters of the models are examined. The effect of changing corrective maintenance models on equipment availability is analyzed. The point availability is the major measure of performance, which is illustrated in the paper by various graphics. Results are obtained using a simulation model created in Visual Basic, and a 95% confidence interval is obtained by multiple repetitions. The models of the impact of repair include Perfect Repair, Minimal Repair, Kijima Types Repair, and Quasi-Renewal Repair. The simulation model is developed to obtain functional approximations to the availability function for each of these models. This research does not provide any analytical solution methods, but it can be used as a tool when measuring the relative error of alternative analytical methods.

Dieulle, Laurence, "Reliability of several component sets with inspections at random times" European Journal of Operational Research 139 (2002) 96-114

This paper considers a random process representing a system of components with constant failure rates and subjected to inspections at times defining a renewal process. An analytical method for calculating the reliability function, its Laplace transform and the mean time to failure is given. These formulas are only computable if the Laplace transform of the inter-arrival law of the renewal process is explicit. The asymptotic behavior of the reliability and the failure rate of the system are studied. The reduced ability to model a variety of distributions, and the intractability when modeling large-scale systems however makes this study less applicable in reality.

Kijima, M. "Some results for repairable systems with general repair" Journal of Applied Probability 26 (1989) 89-102

In this paper Kijima develops general repair models for a repairable system by using the idea of the virtual age process of the system. Two models are constructed depending on how the repair affects the virtual age process. These models are then used to obtain an upper bound for the expected value of the survival function when a general repair is used. The introduction of the Virtual Aging Process is interesting especially considering the large number of references that have been made to this paper. Some knowledge of the survival function makes modeling maintenance policies easier, and the idea of virtual aging can greatly contribute to this endeavor.

Lam, Y., Tony, H, K., "A general model for consecutive-k-out-of-n: F repairable system with exponential distribution and (k-1)-step Markov dependence." European Journal of Operational Research 129 (2001) 663-682

In this paper, a general model for a repairable system in which the lifetime of a component depends on the number of consecutive failed components that precede the

component. The failure and repair time are both exponentially distributed in this model. A transition density matrix is determined, and a general Markovian model is used to obtain some measures of performance such as the availability, the rate of failures and the reliability. The assumption is that a failed component after repair will be "as good as new". This publication has some general ideas that crossover to the other references presented here, and to my own field of research. The proposed methods however lack the ability to solve systems that show failure and repair times that are not exponentially distributed. It exemplifies that there is a serious need for analytical methods that can provide measures of performance for larger scale systems with non-exponentially distributed random variables.

Matis, J.H., Kiffe, T.R., "Stochastic Population Models, a compartmental perspective"

A classic book of Jacquez on compartmental analysis inspired this publication in which stochastic compartmental analysis is reviewed and generating functions are used to obtain many of the results. The theoretical development of the methods used is found in Chapters three and nine, for all practical purposes I focused on chapter three. This chapter explains the use of moments and cumulants when describing single population stochastic models. A standard approach for solving probability functions known as the Kolmogorov differential equations is illustrated. Cumulant functions are shown to be very useful for finding a distributions cumulants by obtaining and solving partial differential equations for associated generating functions. An immigration death model is used to illustrate the theoretical development of the generating function approach. The use of cumulant functions presented in this analysis is applied in the arena of reliability by Matis and Feldman, as described above. I have extended these ideas to find practical application in reliability systems with the support of Dr. T.I.Matis.

Matis, T.I., Feldman, R.M., "Using Cumulant Functions in Queueing Theory" Queueing Systems 40, (2002) 341-353

This publication demonstrates a new procedure for obtaining measures of performance of state-dependent queueing networks. This procedure uses cumulant generating functions and relates these to the intensity functions in the network. The service rate is expressed as polynomial function of the state of the system, from which a partial differential equation of the cumulative generating function is obtained. This partial differential equation then yields a set of ordinary differential equations, which are then solved to obtain the first and second moments of the system with Markovian arrival and service rates. The first and second moments of the random variables in the system provide transient information when describing the system. As the network increases in size this solution method remains tractable, in contrast to the method proposed by Rehmert in the dissertation reviewed above. Cumulant functions have previously been used by Matis and Kiffe to obtain measures of performance of ecological models, such as the spread of honeybees or muskrats. This cumulant derivation procedure is also used in the research of reliability systems subject to non-fatal shocks.

Pham, H. Wang, H. "Imperfect maintenance" European Journal of Operational Research 94 (1996) 425-438

The maintenance of deteriorating systems is often imperfect, as studied by using simulation in the first reference by Cassady and Iyoob. Imperfect maintenance studies using mathematical models for estimating availability functions and reliability have undergone some breakthroughs that are discussed and summarized in this paper. Treatment methods for imperfect maintenance, such as the (p, q) and (p(t), q(t)) rules, the improvement factor method, virtual age method, shock model method, (□ □) rule, are examined and explained. Preventative maintenance policies, such as age-dependent, periodic PM, failures limit policy, sequential PM policy, repair limit policy, and multi-component systems can indicate what model needs to be selected. Although the focus of this paper tends towards modeling maintenance policies to reduce cost, the underlying principals can be very useful when thinking about reliability systems in a more general sense.

Rehmert, I. J., "Time-Dependent Availability Analysis for the Quasi-Renewal Process." Diss. Virginia Polytechnic Institute and University (2000)

This research is based upon the quasi-renewal process proposed by Wang and Pham which is an alternative to the widely studied imperfect repair model, the (p, q) model, proposed by Brown and Proschan. A quasi renewal process can realistically describe the behavior of repairable equipment. The framework provided in this dissertation allows for the description of the time-dependent behavior of this non-homogeneous process. Two equivalent expressions for the point availability of a system with operation intervals and repair intervals that deteriorate according to a quasi-renewal process are constructed. These expressions are used to provide upper and lower bounds on the approximated point availability. Laplace transforms are used to solve the resulting expressions. The quasi renewal function and the point availability function are found for exponential, normal, and gamma operating and repair intervals. It is necessary to truncate the expressions to invert to the time domain to obtain numerical results. The usefulness of this approach seems limited because it does not find exact expressions for all distributions, and the calculations seem to be intractable as the size of the system increases.

Scarsini, M. Shaked, M. "On the value of an item subject to general repair or maintenance" European Journal of Operational Research, Vol.122 (2000) 625-637

This paper introduces and studies finding a practical expression of the monetary value of an item. The model that is constructed takes the repair and the maintenance procedures that are applied to it during its lifetime. The number of repairs and the degree of the repair are taken into account when doing the calculations. The degree of the repair follows the ideas of virtual aging modeling as was presented earlier by Kijima in his virtual aging models. The uncertainty is introduced into the model by the distributions of the inter repair or inter maintenance periods. This paper shows how the virtual aging process can be used to model cost under repair and maintenance procedures.

Sheu, S., Griffith, W.S., "Multivariate Age-Dependent Imperfect Repair" Naval Research Logistics, Vol. 38 (1991) 839-850

This article considers models of systems whose components have dependent life lengths with specific multivariate distributions. Components are repaired according to a corrective maintenance scheme, meaning that components are repaired upon failure. Only two types of repair are considered in this paper: perfect repair, and imperfect repair. The study focuses on a model in which the nature of the repair is age dependent. The model uses the (p(t), q(t)) rule which was earlier described in this reference list in the publication by Wang. An expression for the cumulative hazard function is derived, which can be useful when describing reliability of systems. The paper however lacks any numerical examples that demonstrates finding the hazard function.

Wang, H., "A survey of maintenance policies of deteriorating systems" European Journal of Operational Research 139 (2002) 469-489

This survey summarizes, classifies, and compares various existing maintenance policies for both single-node and multi-node systems. All these models fall into categories such as: Age replacement policy, block replacement policy, periodic preventive maintenance policy, failure limit policy, sequential preventive maintenance policy, repair cost limit policy, repair time limit policy, repair number counting policy, reference time policy, mixed age policy, group maintenance policy, etc. The characteristics, advantages, and drawbacks for each kind of policy are addressed. Maintenance and replacement problems have been studied for the past several decades, and this invited review gives a clear overview of the models used. Wang's research has introduced the concept of "Virtual Age" when modeling Quasi-renewal or imperfect repair.

Based on 9990 repetitions. Failure rate is related to both current and cumulant count

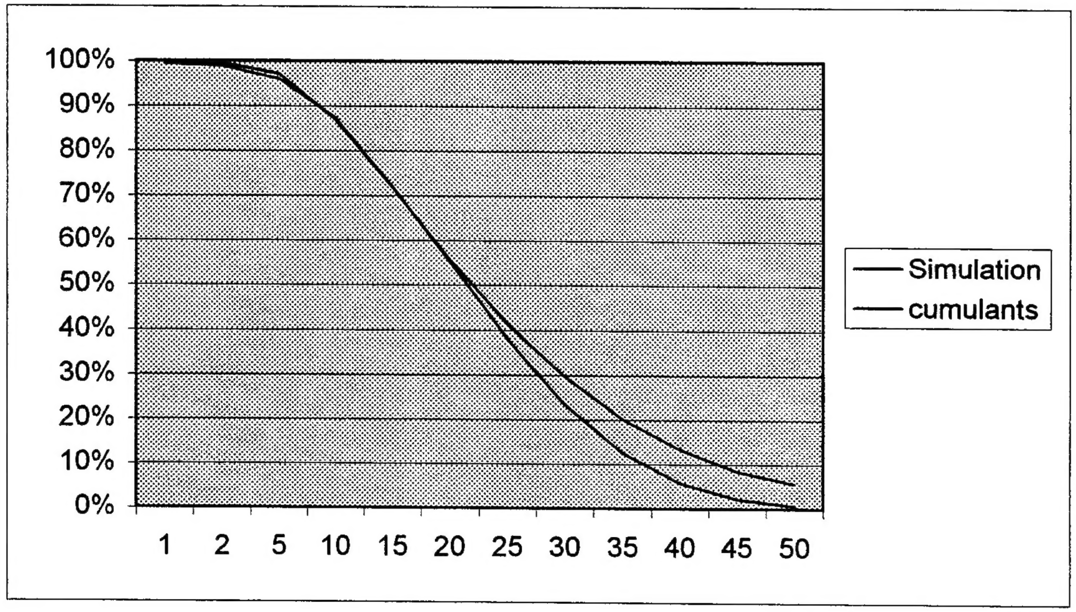
Sum	9984	9948	9708	8686	7182	5570	4126	2970	2006	1336	848	564
Simulation	99.94%	99.58%	97.18%	86.95%	71.89%	55.76%	41.30%	29.73%	20.08%	13.37%	8.49%	5.65%

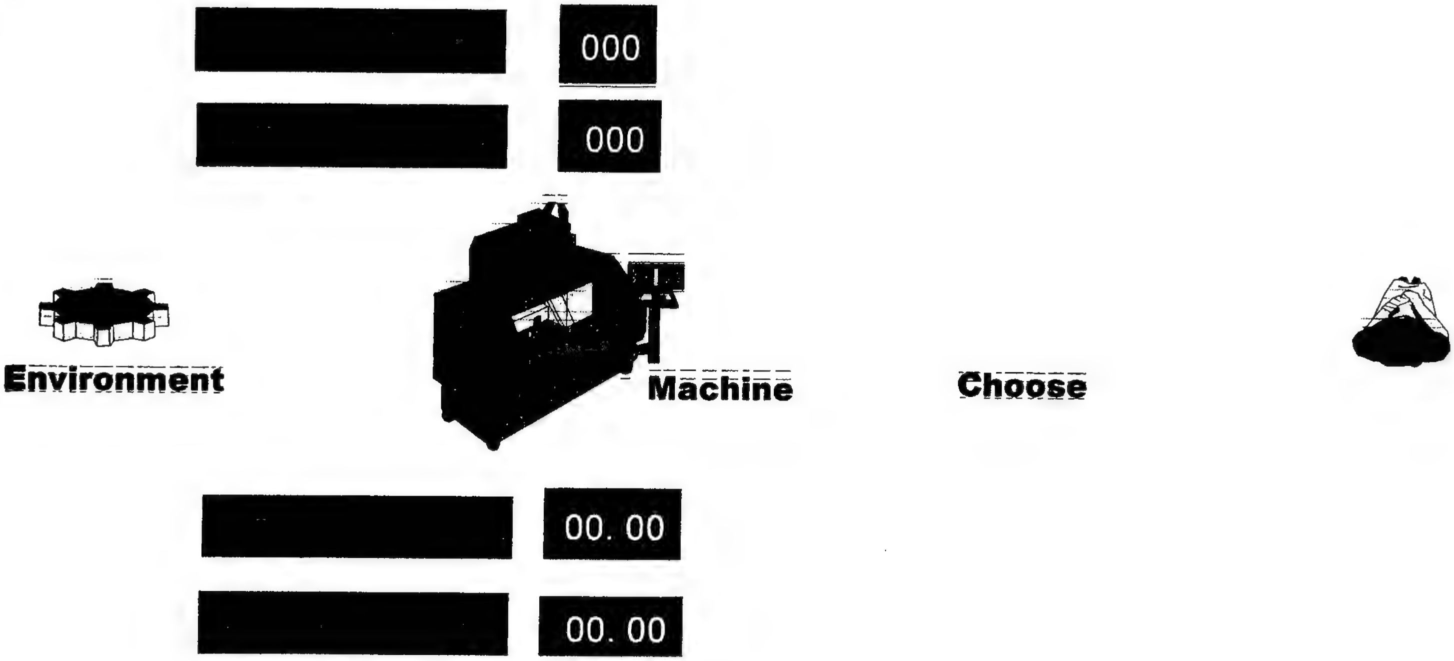
Time	1	2	5	10	15	20	25	30	35	40	45	50
Mathemat	99.46%	98.88%	96.04%	87.28%	72.06%	55.21%	37.98%	23.35%	12.53%	5.74%	2.20%	0.69%

$\lambda = 0.1$
$\mu_2=0.05$
$\mu_3=0.005$
$\alpha=0.008$
$\beta=0.004$

$E[X_3(t)]=\mu_3 x_3+\mu_3 x_3 x_1^2+\mu_3 x_3 x_2 x_1^2$

$E[X_2(t)]=\mu_2 x_2$





Scenario : Normal Run
Replication : 999 of 999
Simulation Time : 0.002319444444 hr

LOCATIONS

Location Name	Scheduled Hours	Capacity	Total Entries	Average Seconds Per Entry	Average Contents	Maximum Contents	Current Contents
Environment	0.002319444444	999999	2	0.000000	0	1	0
Machine	0.002319444444	999999	3	1.226667	0.440719	3	3
Current	0.002319444444	999999	2	1.650000	0.39521	2	1
Choose	0.002319444444	999999	1	0.000000	0	1	1
Write up	0.002319444444	999999	1	0.000000	0	1	1

LOCATION STATES BY PERCENTAGE (Multiple Capacity)

Location Name	Scheduled Hours	% Empty	% Partially Occupied	% Full	% Down
Environment	0.002319444444	100.00	0.00	0.00	0.00
Machine	0.002319444444	64.91	35.09	0.00	0.00
Current	0.002319444444	64.91	35.09	0.00	0.00
Choose	0.002319444444	100.00	0.00	0.00	0.00
Write up	0.002319444444	100.00	0.00	0.00	0.00

RESOURCES

Resource Name	Units	Scheduled Hours	Number of Times Used	Average Seconds Per Usage	% Util
Repair.1	1	0.002319444444	1	2.930000	35.09
Repair.2	1	0.002319444444	1	0.370000	4.43
Repair.3	1	0.002319444444	0	0.000000	0.00
Repair.4	1	0.002319444444	0	0.000000	0.00
Repair.5	1	0.002319444444	0	0.000000	0.00
Repair.6	1	0.002319444444	0	0.000000	0.00
Repair.7	1	0.002319444444	0	0.000000	0.00
Repair.8	1	0.002319444444	0	0.000000	0.00
Repair.9	1	0.002319444444	0	0.000000	0.00
Repair.10	1	0.002319444444	0	0.000000	0.00

```
$HistoryLength = 0;
λ := .8
μ3 := .005
a := .005
b := .005

K[0, 0, 0][t] := 0
M[0, 0, 0] = 1;
(M[i__][t] /; Plus @@ {i} ≥ 4) := 0
(K[i__][t] /; Plus @@ {i} ≥ 4) := 0
```

$$\text{mgf1} := \sum_{l=0}^{3}\sum_{k=0}^{3}\sum_{j=0}^{3} \text{M}[j, k, l] * \frac{\theta_1^j\,\theta_2^k\,\theta_3^l}{j!\,k!\,l!}$$

$$\text{CGF} := \sum_{l=0}^{3}\sum_{k=0}^{3}\sum_{j=0}^{3} \text{K}[j, k, l][t] * \frac{\theta_1^j\,\theta_2^k\,\theta_3^l}{j!\,k!\,l!}$$

```
MGF := Exp[CGF]

deriv1[i_] := D[MGF, {θ1, i}] /. {e^CGF → mgf1}
deriv2[i_] := D[MGF, {θ2, i}] /. {e^CGF → mgf1}
deriv3[i_] := D[MGF, {θ3, i}] /. {e^CGF → mgf1}

g = D[MGF, {t, 1}] /. {e^CGF → mgf1};
```

$$h = \lambda * \left(\sum_{j=1}^{3} \frac{(\theta_1)^j}{j!}\right) * \text{MGF} + \mu_3 * \left(\sum_{j=1}^{3} \frac{(-\theta_3)^j}{j!}\right) * (\text{deriv3}[1]) + a * \left(\sum_{j=1}^{3} \frac{(-\theta_3)^j}{j!}\right) * (\text{deriv3}[1] * \text{deriv1}[1]) + b * \left(\sum_{j=1}^{3} \frac{(-\theta_3)^j}{j!}\right) * (\text{deriv3}[1] * \text{deriv1}[2]) \;/.\; \{e\verb|^|\text{CGF} \to \text{mgf1}\};$$

```
<< momcum.m

UptoOrder[nVar_, ord_] :=
  Rest[Select[Distribute[Table[Range[0, ord], {nVar}], List], (Plus @@ # ≤ ord) &]];
orderList1 = UptoOrder[3, 3];

relationsmod =
  MomCumConvert[#, ForMomentQ → "Y", CenteredQ → "N", MomentSymbol → M, CumulantSymbol → K] & /@
   orderList1;

relations := relationsmod /. {K[i__] → K[i][t]}
momcumrule = relations /. {Equal → Rule};
```

```
eq1[i_, j_, k_] := h /. {θ1 → Evaluate[i * θ1], θ2 → Evaluate[j * θ2], θ3 → Evaluate[k * θ3]};
dum1[i_, j_, k_] := Coefficient[Evaluate[
     eq1[Sign[i], Sign[j], Sign[k]]], θ1^i θ2^j θ3^k] /. momcumrule;
eq2[i_, j_, k_] := g /. {θ1 → Evaluate[i * θ1], θ2 → Evaluate[j * θ2], θ3 → Evaluate[k * θ3]};
dum2[i_, j_, k_] := Coefficient[Evaluate[
     eq2[Sign[i], Sign[j], Sign[k]]], θ1^i θ2^j θ3^k] /. momcumrule;

bilbo = MapThread[dum1, Transpose[orderList1]];

bilbo2 = MapThread[dum2, Transpose[orderList1]];

Shitake = Table[bilbo2[[i]] == bilbo[[i]], {i, 1, 19}];

Bonzai = Table[
   K[Part[orderList1, i, 1], Part[orderList1, i, 2], Part[orderList1, i, 3]]'[t], {i, 1, 19}];
neweqns = Solve[Shitake, Bonzai];
neweqmod = neweqns /. {Rule → Equal};

Samuri = Table[K[Part[orderList1, i, 1], Part[orderList1, i, 2], Part[orderList1, i, 3]][0] == 0,
   {i, 1, 19}];

Monkey = ReplacePart[Samuri, K[0, 0, 1][0] == 1, 1];

Joy = Join[First[neweqmod], Monkey];
Pokemon =
  Table[K[Part[orderList1, i, 1], Part[orderList1, i, 2], Part[orderList1, i, 3]], {i, 1, 19}];

rs = NDSolve[Joy, Pokemon, {t, 0, 50}, MaxSteps → 10000]

{{K[0, 0, 1] → InterpolatingFunction[{{0., 50.}}, <>],
  K[0, 0, 2] → InterpolatingFunction[{{0., 50.}}, <>],
  K[0, 0, 3] → InterpolatingFunction[{{0., 50.}}, <>],
  K[0, 1, 0] → InterpolatingFunction[{{0., 50.}}, <>],
  K[0, 1, 1] → InterpolatingFunction[{{0., 50.}}, <>],
  K[0, 1, 2] → InterpolatingFunction[{{0., 50.}}, <>],
  K[0, 2, 0] → InterpolatingFunction[{{0., 50.}}, <>],
  K[0, 2, 1] → InterpolatingFunction[{{0., 50.}}, <>],
  K[0, 3, 0] → InterpolatingFunction[{{0., 50.}}, <>],
  K[1, 0, 0] → InterpolatingFunction[{{0., 50.}}, <>],
  K[1, 0, 1] → InterpolatingFunction[{{0., 50.}}, <>],
  K[1, 0, 2] → InterpolatingFunction[{{0., 50.}}, <>],
  K[1, 1, 0] → InterpolatingFunction[{{0., 50.}}, <>],
  K[1, 1, 1] → InterpolatingFunction[{{0., 50.}}, <>],
  K[1, 2, 0] → InterpolatingFunction[{{0., 50.}}, <>],
  K[2, 0, 0] → InterpolatingFunction[{{0., 50.}}, <>],
  K[2, 0, 1] → InterpolatingFunction[{{0., 50.}}, <>],
  K[2, 1, 0] → InterpolatingFunction[{{0., 50.}}, <>],
  K[3, 0, 0] → InterpolatingFunction[{{0., 50.}}, <>]}}
```

```
K[0, 0, 1][2] /. rs
K[0, 0, 1][4] /. rs
K[0, 0, 1][6] /. rs
K[0, 0, 1][8] /. rs
K[0, 0, 1][10] /. rs
K[0, 0, 1][12] /. rs
K[0, 0, 1][14] /. rs
K[0, 0, 1][16] /. rs
K[0, 0, 1][18] /. rs
K[0, 0, 1][20] /. rs
K[0, 0, 1][22] /. rs
K[0, 0, 1][24] /. rs
K[0, 0, 1][26] /. rs
Plot[Evaluate[{K[1, 0, 0][t]} /. rs], {t, 0, 50},
 PlotRange → {0, 10}, AxesLabel → {"t", "E[X1(t)]"}]
Plot[Evaluate[{K[0, 1, 0][t]} /. rs], {t, 0, 50},
 PlotRange → {0, 5}, AxesLabel → {"t", "E[X2(t)]"}]
Plot[Evaluate[{K[0, 0, 1][t]} /. rs], {t, 0, 50},
 PlotRange → {0, 1}, AxesLabel → {"t", "E[X3(t)]"}]
Plot[Evaluate[{K[2, 0, 0][t]} /. rs], {t, 0, 50},
 PlotRange → {0, 10}, AxesLabel → {"t", "Var[X1(t)]"}]
Plot[Evaluate[{K[0, 2, 0][t]} /. rs], {t, 0, 50},
 PlotRange → {0, 5}, AxesLabel → {"t", "Var[X2(t)]"}]
```

{0.966056}

{0.858759}

{0.667377}

{0.430791}

{0.219442}

{0.0838102}

{0.0228014}

{0.00419834}

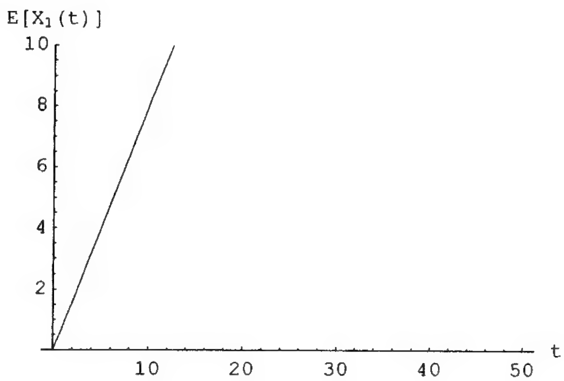
{0.000497058}

{0.0000359521}

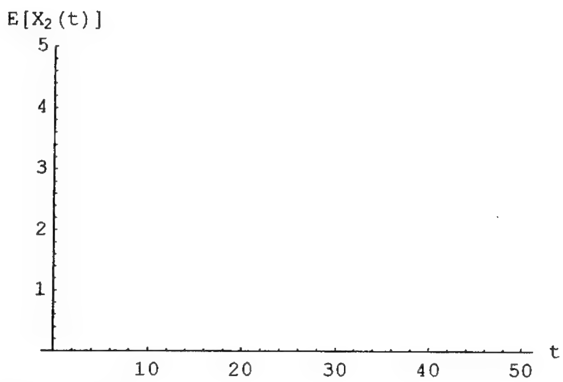
$\{1.50969\times10^{-6}\}$

$\{3.50847\times10^{-8}\}$

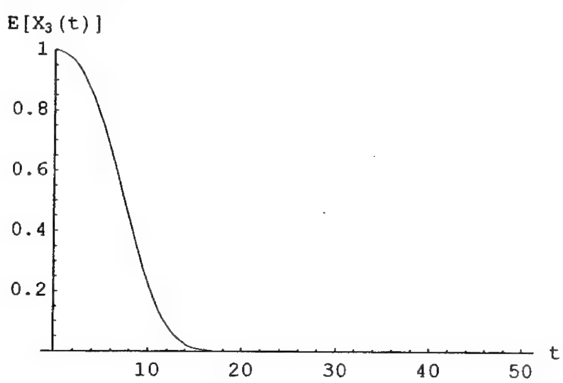
$\{4.46547\times10^{-10}\}$



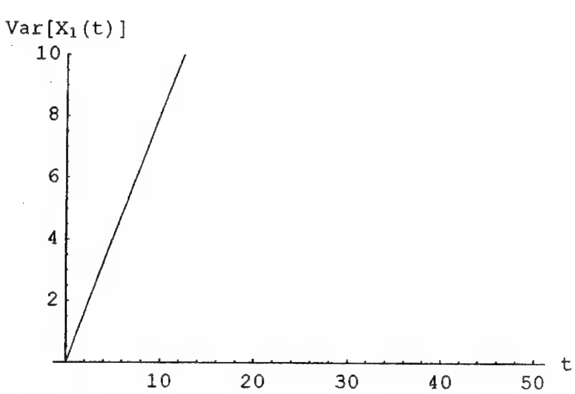
- Graphics -



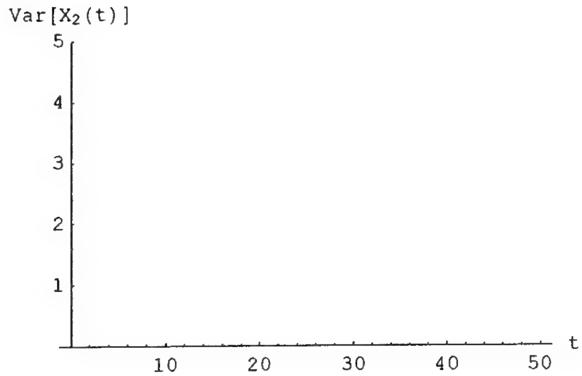
- Graphics -



- Graphics -



- Graphics -



- Graphics -

```
(* <<CumPlot.m *)
(*Nat=6;
 Time=40;
 Z=MapThread[K,IdentityMatrix[Nat]];
 Z=Z/.{K[q__]→K[q][t]}
    For[i=1,i<Nat+1,Plot[Evaluate[Take[Z,{i,i}]/.rs],{t,0,Time}];i++]*)
```

Reliability Systems Subject to Non-Fatal Shocks

Timothy I. Matis and Martijn Kolloffel
Department of Industrial Engineering
New Mexico State University
P.O. Box 30001, MSC 4230
Las Cruces, NM 88003-8001
tmatis@nmsu.edu,mkollof@nmsu.edu

Abstract

The reliability modeling of numerous physical systems is critical in the prevention of system failure. In many instances, the failure rate of system components is a function of non-fatal shocks or stresses to the system that occur at discrete points in time. These shocks are assumed to be identical and reparable, and they impact the failure rate of the system in a non-linear fashion via the cumulative and current number of shocks. In this paper, we demonstrate the application of the cumulant derivation procedure to this reliability system in a Markovian environment. This approach utilizes a truncated cumulant generating function to generate a set of ordinary differential equations whose numerical solution approximates the reliability function. These approximations are obtained under various truncation levels whereby this approach is shown to be tractable for large systems.

1. Introduction

In this paper, we consider a single component operating in an environment of non-fatal reparable shocks. These shocks may represent a wide variety of events, including the failure of other system components, instantaneous system stresses, or the states of a compartmentalized wear process. The shocks are assumed to be homogenous and their arrival is governed by a stationary Poisson process. They are repaired according to an exponentially distributed infinite server repair process that begins immediately upon shock arrival whose rate is a function of the total number of cumulative shocks. The failure of the component is Poisson distributed with the rate being a function of both the current number of unrepaired shocks and the total number of shocks that have ever been received by the system. In other words, each shock has both an immediate reparable effect and a permanent weakening effect on the system. Let $X_1(t)$ and $X_2(t)$ be integer-valued random variables taking values in $[0,1,..,\infty]$ that denote the cumulative and current number of shocks at time t respectively. Let $X_3(t)$ be an integer-valued random variable taking values $[0,1]$ that represents the state of the component at time t, where $X_3(t)=1$ denotes a functioning component and $X_3(t)=0$ denotes a failed component. All possible unit changes that may occur in the state of the system in a small interval of time are contained in the set B and the corresponding state-dependent intensity (rate) functions will be denoted as $f_{(b1,b2,b3)}(X_1,X_2,X_3)$ for $(b_1,b_2,b_3)\in B$. This system is graphically depicted in Figure 1.

We assume that initially the component is operational, $X_3(0)=1$, and no shocks have been received $X_1(0)=X_2(0)=0$. The intensity function corresponding to component failure, $f_{(0,0,-1)}(X_1,X_2,X_3)$, is specified as a nonlinear function of the cumulative and current number of shocks, $X_1(t)$ and $X_2(t)$, and represents the damage process previously described. Our primary interest is in approximating the reliability function of the component, i.e. the expected value of $X_3(t)$, for all $t \geq 0$ using truncated cumulant generating functions. These approximations are compared to simulated values for several systems under various intensity function specifications and truncation levels.

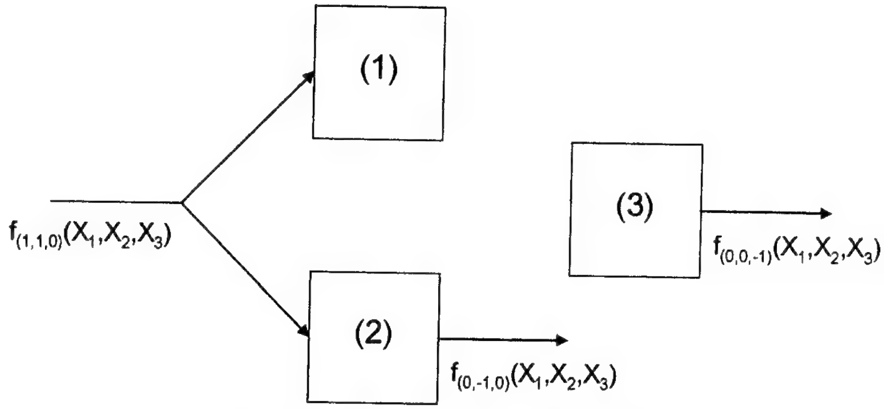


Figure 1: Graphical Representation of Reliability System

2. Cumulant Derivation Procedures

Let X(t)=(X_1(t),X_2(t),,X_3(t),) be a random vector of the system state at time t. It follows that X(t) forms a Markov process with an absorbing state that denotes component failure. While such a process may be solved exactly using convention means, i.e. Kolomogorov Equations, such an approach is intractable for large systems or those with an infinite state space. As an alternative, the cumulants of the state-distribution of X(t) may be approximated using a truncated generating functions. These cumulant measures correspond directly to the common measure of mean, variance, covariance, skewness, etc. of the state distribution. Previous investigations into the nature of cumulant functions by Kendall[1][2] and Smith[3] reveal several interesting properties. In particular, the cumulants of a multivariate normal distribution greater than the second order are null, and the marginal cumulants of a Poisson distribution are equal. This and other properties of cumulants are exploited by this approximation procedure. This section contains only a brief overview of the cumulant derivation procedure based on the full development found in Matis and Feldman [4][5], Matis [6], and Matis and Kiffe [7].

Let $M(\theta_1,\theta_2,\theta_3,t)$ be the multivariate moment generation function of X(t) defined in the usual manner, and let $K(\theta_1,\theta_2,\theta_3,t)$ be the multivariate cumulant generating function defined as

$$K(\theta_1,\theta_2,\theta_3,t) = \sum_{a_1,\ldots,a_n \in N^+} \frac{k_{a_1,\ldots,a_n}(t)\theta_1^{a_1}\cdots\theta_n^{a_n}}{a_1!\cdots a_n!}$$

where N^+ denoted the set of non-negative integers. The joint cumulants $k_{a_1,\ldots,a_n}(t)$ of the system are defined as functions of the individual moments through the relationship between the generating functions, i.e.

$$K(\theta_1,\theta_2,\theta_3,t) = \ln\left(M(\theta_1,\theta_2,\theta_3,t)\right). \tag{1}$$

The moment generating function of X(t) related to the polynomial intensity functions of the system through a partial differential equation. This relationship was investigated by Bartlett[8] and Bailey[9] and thereby dubbed the "Random Variable Technique". While the specification of the partial differential equation of the moment generating is almost immediate, finding a solution is usually computationally intractable. The cumulant derivation procedure involves substituting a truncated cumulant generating function for the moment generating function into the partial differential equation according to Eq. (1). A set of approximating ordinary differential equations is obtained upon expanding the partial derivatives, substituting Taylor series expansions for the exponential terms, and equating the coefficients of

unique combinations of the θ_i's. The accuracy of these approximations is dependent upon the specification and order of the polynomial intensity functions and the level of cumulant truncation. Previous investigations, Matis[6], have shown that truncation at the 3rd order is generally sufficient for "good" approximations of the first order cumulants (mean), while that at the 4th order is sufficient for the marginal second order cumulants (variance). Further developments of the cumulant derivation procedure will be described in a reliability context.

3. Cumulant-Based Analysis of the Non-Fatal Shock Process

In this section, we demonstrate the application of cumulant-based procedures to the non-fatal shock process described in the introduction of this paper, see Figure 1. This will be shown for one instance of the problem under a unique set of intensity functions. Let the intensity function of the system be defined as

$$f_{(1,1,0)}(X_1,X_2,X_3) = \lambda$$
$$f_{(0,-1,0)}(X_1,X_2,X_3) = \mu_2 X_1^2(t)X_2(t)$$
$$f_{(0,0,-1)}(X_1,X_2,X_3) = \mu_3(X_3(t)+X_1(t)X_2^2(t)\,X_3(t))$$

where λ=.10, μ_2=.05, and μ_3=.025. In other words, the rate of repair is dependent upon the cumulative number of shocks and the failure rate of the component is dependent on both the current and cumulative number of shocks, both in an increasing manner. A partial differential equation of the moment generating function of X(t) is found using the "Random Variable Technique" as

$$\frac{\partial M(\theta_1,\theta_2,\theta_3,t)}{\partial t} = \lambda e^{(\theta_1+\theta_2-1)}\frac{\partial M(\theta_1,\theta_2,\theta_3,t)}{\partial\theta_1} + \mu_2 e^{(-\theta_2-1)}\frac{\partial^3 M(\theta_1,\theta_2,\theta_3,t)}{\partial\theta_1^2\partial\theta_2}$$
$$+\mu_3 e^{(-\theta_3-1)}\left(\frac{\partial M(\theta_1,\theta_2,\theta_3,t)}{\partial\theta_3} + \frac{\partial^4 M(\theta_1,\theta_2,\theta_3,t)}{\partial\theta_1\partial\theta_2{}^2\partial\theta_3}\right). \tag{2}$$

An mth order truncated cumulant generating function $K_m(\theta_1,\theta_2,\theta_3,t)$ and a Taylor series expansion of the exponential terms is substituted into Eq.(2) yielding the expression

$$\frac{\partial e^{K_m(\theta_1,\theta_2,\theta_3,t)}}{\partial t} = \lambda\left(\sum_{i=1}^{\infty}\frac{(\theta_1+\theta_2)^i}{i!}\right)\frac{\partial e^{K_m(\theta_1,\theta_2,\theta_3,t)}}{\partial\theta_1} + \mu_2\left(\sum_{i=1}^{\infty}\frac{(-\theta_2)^i}{i!}\right)\frac{\partial^3 e^{K_m(\theta_1,\theta_2,\theta_3,t)}}{\partial\theta_1^2\partial\theta_2}$$
$$+\mu_3\left(\sum_{i=1}^{\infty}\frac{(-\theta_3)^i}{i!}\right)\left(\frac{\partial e^{K_m(\theta_1,\theta_2,\theta_3,t)}}{\partial\theta_3} + \frac{\partial^4 e^{K_m(\theta_1,\theta_2,\theta_3,t)}}{\partial\theta_1\partial\theta_2{}^2\partial\theta_3}\right). \tag{3}$$

Expanding the partial derivatives, converting moments to cumulants (Smith[3]), and equating the coefficients of like polynomial terms on the right and left hand side of Eq. (3) yields a closed set of $\left(\frac{1}{3!}\prod_{i=1}^{3}(i+m)\right)-1$ ordinary differential equations. The size of the generated sets of ordinary differential equation does not permit their demonstration in this paper, yet the number of such equation is 9 under a truncation level of m=2, 19 under m=3, and 34 under m=4. Approximations to the low order cumulants are found upon numerically solving these sets of ordinary differential equations.

4. Numerical Results

The sets of ordinary differential equations generated from Eq. (3) were numerically solved using the mathematical software Mathematica® under the truncation levels m=2, 3, 4. These were then compared to simulated point estimates based on 10,000 replications using the software ProModel®. The initial

conditions for the process have all cumulants set equal to zero except for $k_{0,0,1}(0)$, which corresponds directly to the reliability of the component at time 0, is set to one. As previously noted, truncating the cumulants at m=2 implies that the state-distribution is normally distributed. Increasing the level of truncation to m=3 brings in skewness and m=4 brings in kurtosis, in addition to the effects of the higher-order cross cumulants. The cumulant approximations for the reliability of the component, i.e. $R(t)=E[X_3(t)]$, is given in Figure 2.

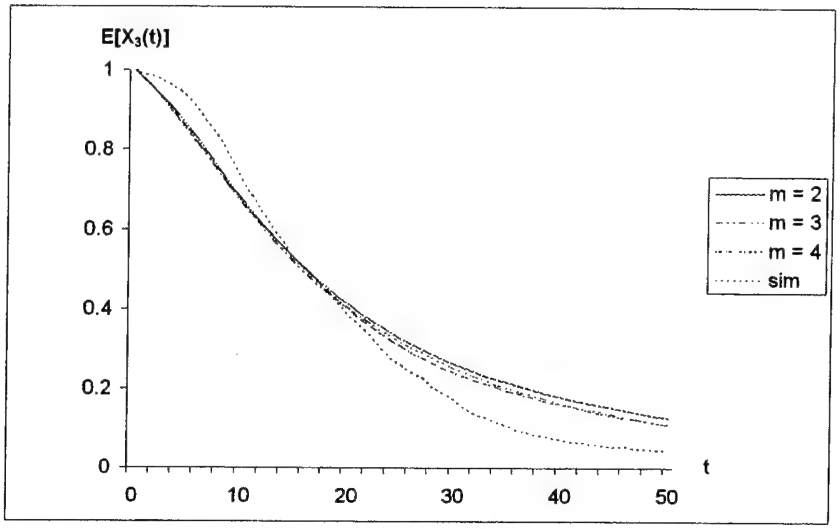


Figure 2: Graph of $R(t) = E[X_3(t)]$ for Varying Truncation Levels

The approximation of $R(t)=E[X_3(t)]$ is relatively tight between all values of m and close to the simulated value. This result is promising as the reliability function for systems subject to similar non-fatal shock processes may be well approximated using small values of m. As such, this result provides evidence that approach may be extended to similar large-scale networks in a computationally efficient manner. Though not of primary interest in this paper, the differences in approximations of the variance of the current number of shocks, $Var[X_2(t)]$, between truncation levels is noteworthy and is given in Figure 3.

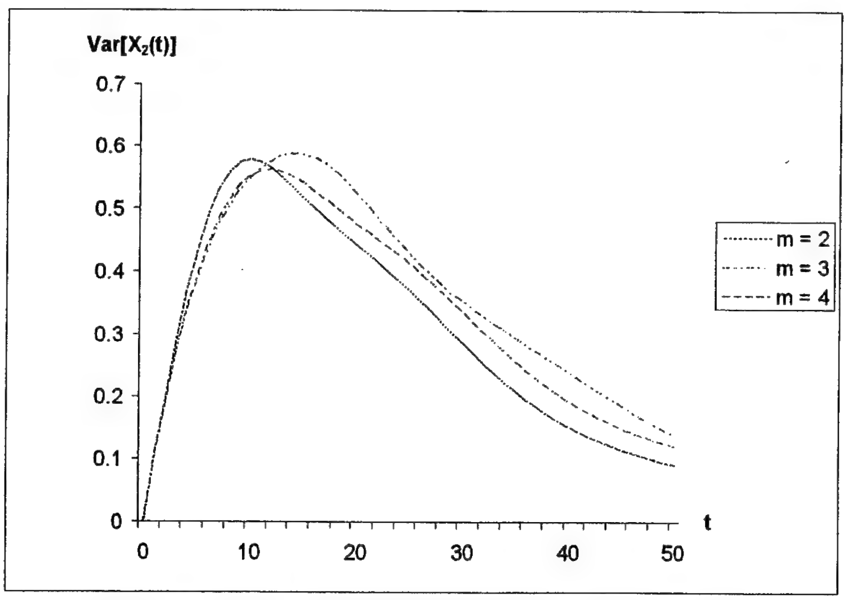


Figure 3: Graph of $Var[X_2(t)]$ for Varying Truncation Levels

The specified model was then evaluated under the parameter specification λ=.10, μ_2=.05, and μ_3 =.005 yielding the graph in Figure 4. Comparing the similarity of the cumulant approximations for various truncation levels in both Figures 2 and 4 provides evidence that the precision of the reliability function approximations do not significantly vary with the rate of system failure. Comparing these approximations to simulations, however, provides evidence that the accuracy of the approximations increases as the rate of system failure decreases.

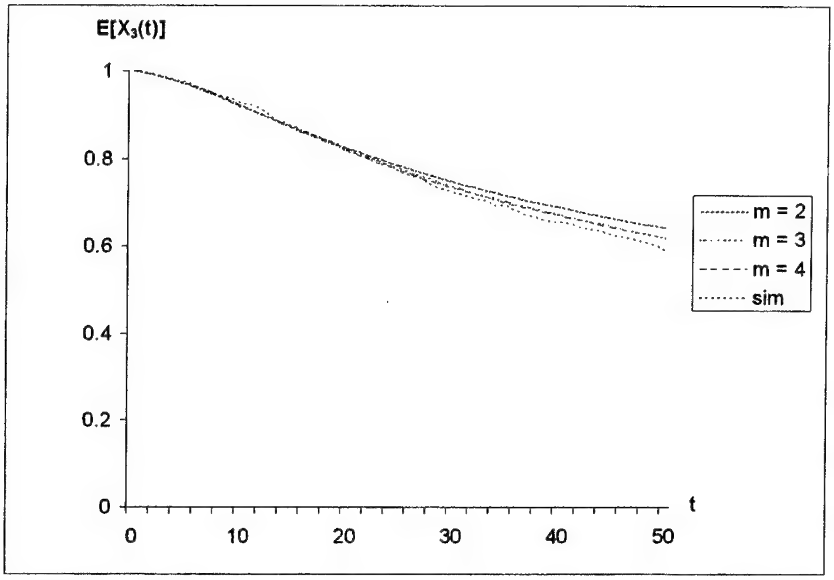


Figure 4: Graph of R(t) = $E[X_3(t)]$ for Varying Truncation Levels

The polynomial intensity function of the model were then redefined as

$$f_{(1,1,0)}(X_1,X_2,X_3) = \lambda$$
$$f_{(0,-1,0)}(X_1,X_2,X_3) = \mu_2 X_1^3(t)X_2(t)$$
$$f_{(0,0,-1)}(X_1,X_2,X_3) = \mu_3(X_3(t)+X_1^2(t)X_2^3(t)\ X_3(t))$$

increasing the dependency of the failure and repair rates on the state of the network. The graph of the reliability function is given in Figure 5. This deviation of the cumulant approximation under *m=2* provides evidence that 2[nd] order truncation is not sufficient for approximating the cumulants of systems that are strongly state-dependent. This observed result is consistent with the previously stated properties of cumulants, i.e. truncation at *m=2* assumes a multivariate normal state distribution. This normal assumption clearly does not hold for systems with strong state-dependency in which skewness is clearly present.

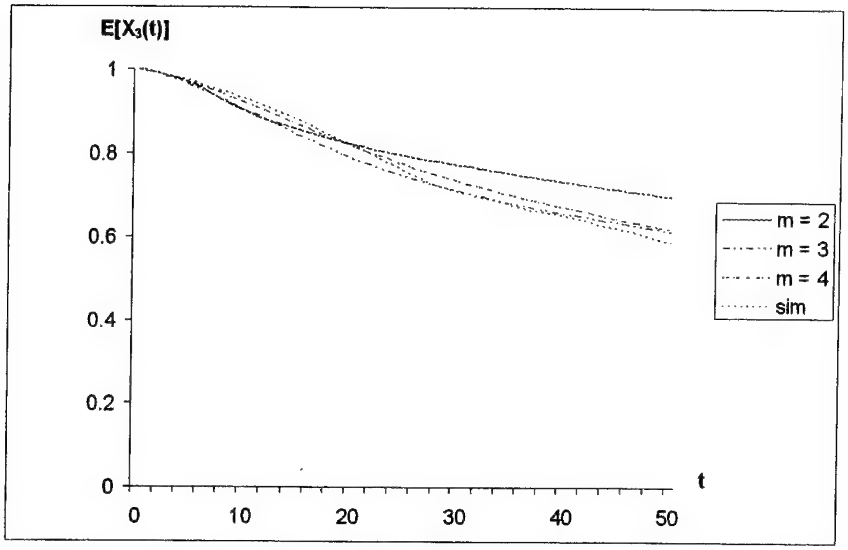


Figure 5: Graph of R(t) = $E[X_3(t)]$ for Varying Truncation Levels

5. Conclusions

In this paper, we have shown the application of cumulant derivation procedures to a reliability system subject to non-fatal shocks, i.e. a state-dependent reliability system. The effect of various truncation levels on the accuracy of the reliability function approximation was demonstrated for various parameters and intensity functions. The similarities between these approximations provide insight into the expandability of the approach to large, complex systems. A copy of the Mathematica® computational routines used to set up and solve the approximating set of ordinary differential equations may be obtained from the authors upon request.

6. Acknowlegements

This work was partially supported by the National Aeronautics and Space Association under Grant# NAG9-1492 and the Office of Naval Research under Grant# N00014-02-1-1040.

References

[1] Kendall, M.G., 1940, "The derivation of multivariate sampling formulae from univariate formulae by symbolic operation", *Annals of Eugenics*, **10**, 392-402

[2] Kendall, M.G., 1994, *Advanced Theory of Statistics*, Halstead Press, New York.

[3] Smith, P.J., 1995, "A recursive formulation of the old problem of obtaining moments from cumulants and vice versa, *The American Statistician*, **49**, 217-218.

[4]Mati,s T.I. and Feldman, R.F., 2002, "Using Cumulant Functions in Queueing Theory", *Queueing System,* **40**, 341-353

[5]Matis, T.I. and Feldman, R.F., 2001, Transient Analysis of State-Dependent Queueing Networks via Cumulant Functions, *Journal of Applied Probability*, **38**, 841-859.

[6]Matis, T.I., 2001, *Using Cumulant Funtions in Queueing Theory, Dissertation*, Texas A&M University

[7]Matis, J.H. and Kiffe, T.R., 2000, *Stochastic Population Models-A Compartmental Perspective*, Springer, New York.

[8] Bartlett, M.S., 1955, *An Introduction to Stochastic Processes*, Cambridge University Press

[9] Bailey, N.T.J., 1964, *Elements of Stochastic Processes*, John Wiley, New York.

Reliability Systems Subject to Non-Fatal Shocks

Timothy I. Matis†, Martijn Kolloffel†‡
Department of Industrial Engineering
Center for Stochastic Modeling
New Mexico State University

† Supported in part by NASA grant # NAG9-1547
‡ Supported in part by ONR grant #N00014-02-1-1040

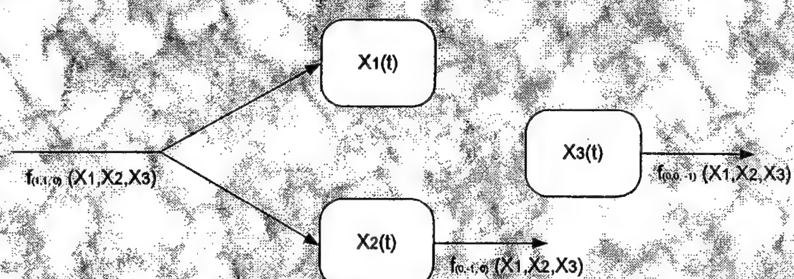
Problem Definition

- Single unit system operating in non-fatal shock environment
- Shocks are homogenous with unit damage
- Each shock causes two types of system damage
  - Reparable damage – transient weakening
  - Irreparable damage – permanent weakening
- The rate of system failure is a non-linear function of the current and cumulative level of damage to the system

Model

- $X_1(t)$ = Cumulative damage
- $X_2(t)$ = Current damage
- $X_3(t)$ = Component Status (1-working, 0-failed)
- Shocks are Poisson distributed with intensity $f_{1,1,0}(X_1, X_2, X_3)$
- Repairs are Exponentially distributed with intensity $f_{0,-1,0}(X_1, X_2, X_3)$
- System Failure is Exponentially distributed with intensity $f_{0,0,-1}(X_1, X_2, X_3)$
- Objective: Find the reliability function $R(t) = \Pr(T_{failure} \geq t) = E[X_3(t)]$ for the system

Graphical Representation



Example

- $f_{1,1,0}(X_1, X_2, X_3) = \lambda$
- $f_{0,-1,0}(X_1, X_2, X_3) = \mu_2 X_1^{\,2} X_2$
- $f_{0,0,-1}(X_1, X_2, X_3) = \mu_3 X_3(1+X_1 X_2^{\,2})$

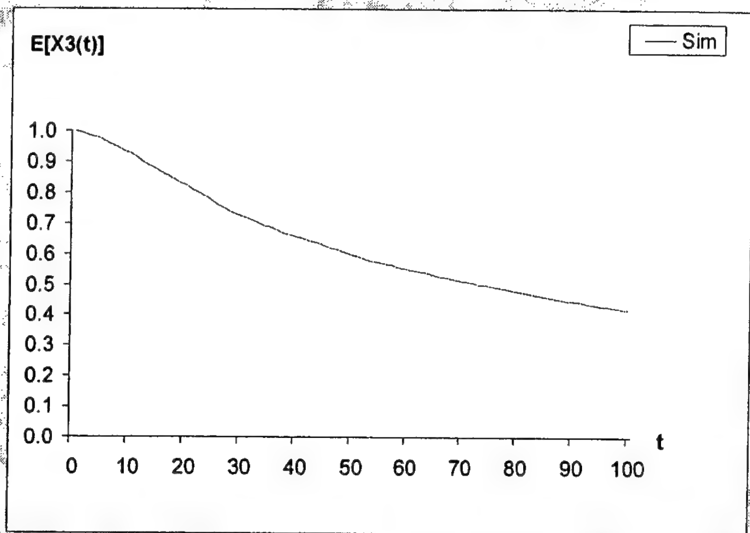
with the numerical parameters:
λ=0.10, μ_2=0.05, μ_3=0.005

Simulated Output

- Stochastic event simulation using ProModel®
- R.V.'s $X_1(t)$, $X_2(t)$, $X_3(t)$ are nodes in the simulation
- Time until failure, i.e. $X_3(t)=0$, is recorded
- Point estimate of $E[X_3(t)]$ is based on 10,000 replications
  - Approx. 300,000 replications are needed to achieve a 95% confidence interval on $E[X_3(t)]$ with a half width of .05
- Unique seed values were assigned for each replication
- Point estimates are graphically represented as a reliability function

Center for Stochastic Modeling

Graph of R(t)



Center for Stochastic Modeling

Deterministic Solution

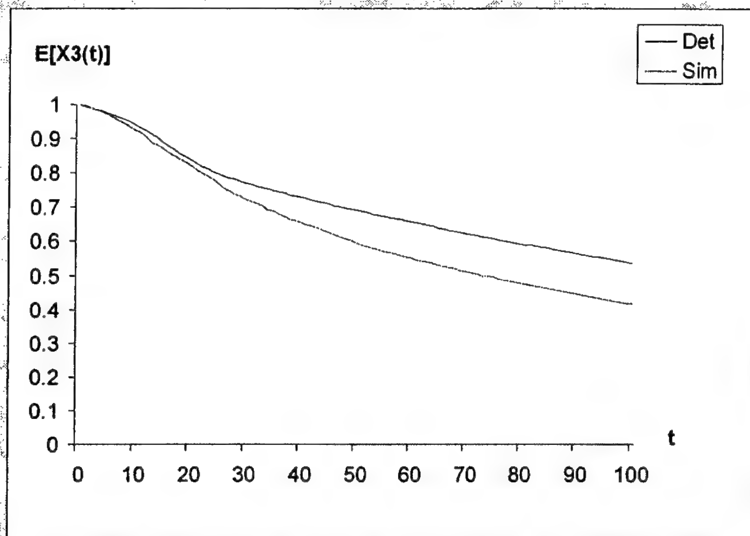
$$\frac{\partial}{\partial t}X_1 = 0.1$$

$$\frac{\partial}{\partial t}X_2 = 0.1 - 0.05X_1^2 X_2$$

$$\frac{\partial}{\partial t}X_3 = -0.005X_3(1 + X_1 X_2^2)$$

$$X_1(0) = X_2(0) = 0, X_3(0) = 1$$

Plot of R(t)



Center for Stochastic Modeling

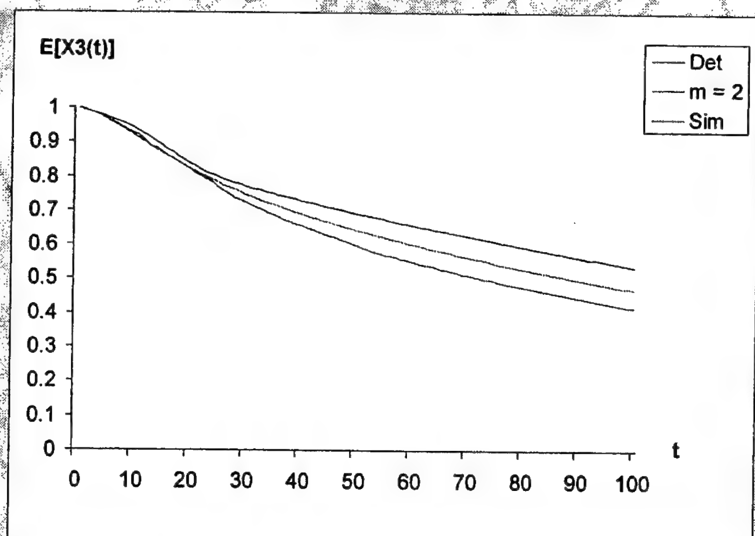
Cumulant-Based Solutions

- Find a partial differential equation of the cumulant generating function of the multivariate distribution of X_1, X_2, X_3 via the "Random Variable Technique" Bartlett (1955).
- PDE is non-linear – hard to solve directly
- As an approximation, truncate cumulants and equate terms of the pde to form a closed set of ordinary differential equations.

Truncation of Cumulants above 2nd order (m=2)

- Truncation above m=2 is equivalent to assuming a multivariate normal state distribution for X_1, X_2, X_3
- This generates a set of 9 differential equations describing $E[X_i]$, $Var[X_i]$ and $Cov[X_i,X_j]$

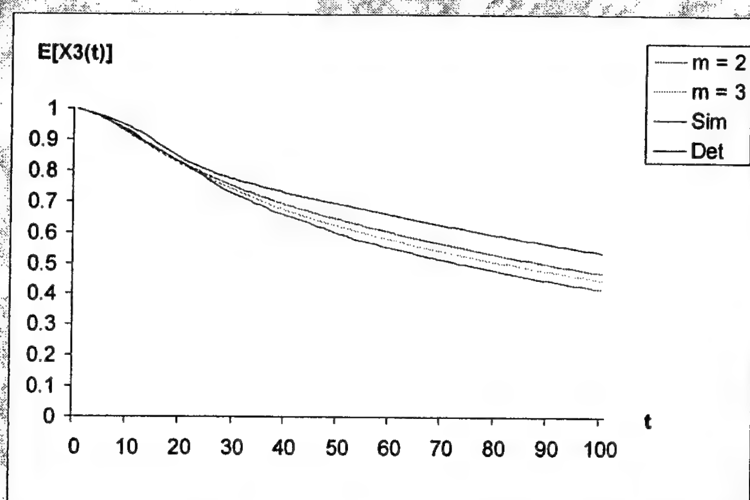
Plot of R(t)



Truncation Level m=3

- Truncation above the 3rd order assumes the multivariate state distribution is fully specified by the mean, variance, and skewness of X_1, X_2, X_3 (and all cross measures below 3rd order)
- This generates a set of 19 ODE's

Plot of R(t)

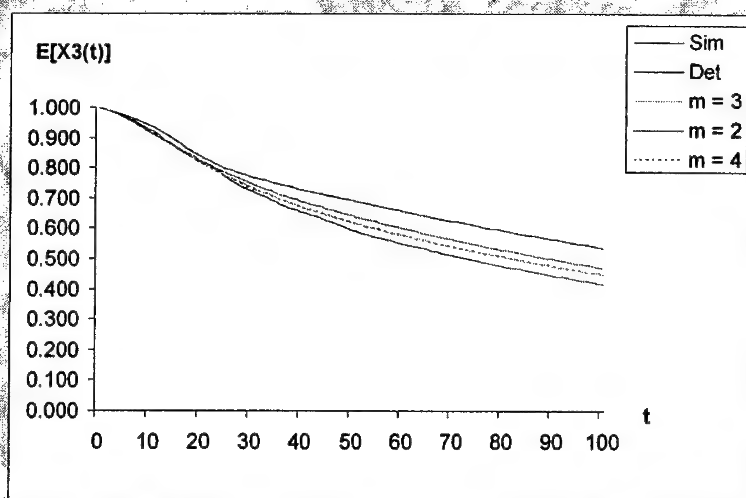


7

Truncation Level m=4

- Truncation above the 4[rd] order assumes the multivariate state distribution is fully specified by the mean, variance, skewness, and kurtosis of X_1, X_2, X_3 (and all cross measures below 4[rd] order)
- This generates a set of 34 ODE's

Plot of R(t)

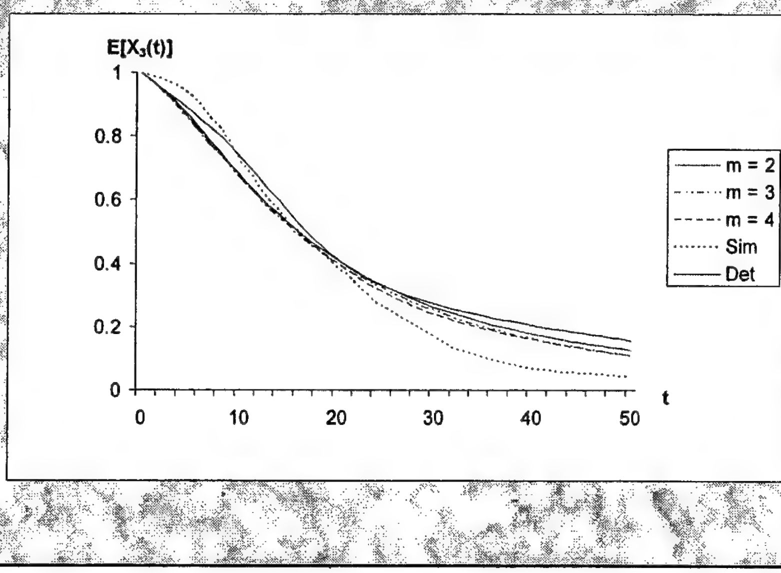


Example – Increased Failure Rate

- $f_{1,1,0}(X_1, X_2, X_3) = \lambda$
- $f_{0,-1,0}(X_1, X_2, X_3) = \mu_2 X_1^2 X_2$
- $f_{0,0,-1}(X_1, X_2, X_3) = \mu_3 X_3 (1+X_1 X_2^2)$

with the numerical parameters:
$\lambda=0.10$, $\mu_2=0.05$, $\mu_3=0.025$

Plot of R(t)



Center for Stochastic Modeling

Example – Modified State-Dependency

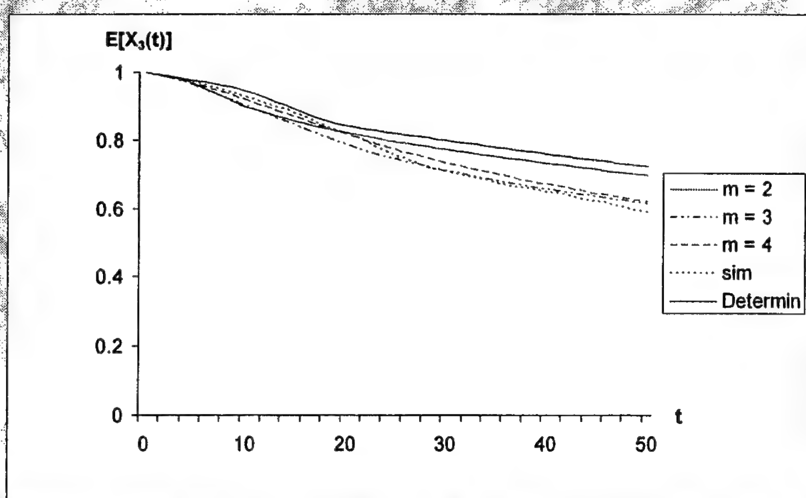
- $f_{1,1,0}(X_1, X_2, X_3) = \lambda$
- $f_{0,-1,0}(X_1, X_2, X_3) = \mu_2 X_1^3 X_2$
- $f_{0,0,-1}(X_1, X_2, X_3) = \mu_3 X_3(1+X_1^2 X_2^3)$

with the numerical parameters:

$\lambda=0.10, \mu_2=0.05, \mu_3=0.005$

Center for Stochastic Modeling

Plot of R(t)



Conclusion

- Cumulant approximations are quick, efficient, and accurate
- Further investigations:
  - Quantifying the effect of specifying various underlying state distributions on approximation accuracy
  - Extension of model to networks

Questions ??

Timothy I. Matis, Ph.D
tmatis@nmsu.edu

Center for Stochastic Modeling
http://engr.nmsu.edu/~csm

NMSU Industrial Engineering
http://engr.nmsu.edu/~ie

Research Log
Investigations into the Application of Cumulant Functions in Operations Research and Stochastic Modelling

Printed Name and Signature Martyr Kolloffel [signature]

For the week ending September 19th 2003

☑ By checking this box, I declare that I have devoted the equivalent of 20 hours of effort to this research project this week.

Research Activities:

Library Research: "Analysis of Equipment Availability under varying Corrective Maintenance Model by Cassady & Iyoob.
- "Some results for repairable systems with general repair" by Kijma, M.

Model formulation: A gear box in which the rate of failure is dependent on the state at other individual components.

Key Research Findings:

- shocks arrive according to Poisson to component
- Components are in state of
  - no wear
  - light wear
  - Moderate wear
  - Severe wear.
- look at: inspection, reliability, sensors, componen

Research Log
Investigations into the Application of Cumulant Functions in Operations Research and Stochastic Modelling

Printed Name and Signature Martyn Kolloffel *[signature]*

For the week ending September 26th 2003

☑ By checking this box, I declare that I have devoted the equivalent of 20 hours of effort to this research project this week.

Research Activities: - Lam, Y: "A general model for consecutive-K-out-of

Library research:- Rehmert, I.J, "Time dependent availabili Analysis for the Quasi-Renewal process."

Model formulation: Aging of component as a continues function over time. Component ages as the result of stresses or shocks at discrete points in time

Key Research Findings:

- Reliability → Define performance function that desribes the performance of the component ⇒ 2nd degree polynomial (non-linear).
- Shock Model → Components are monitored at disrete points in time, or a discrete aging process.

Research Log
Investigations into the Application of Cumulant Functions in Operations Research and Stochastic Modelling

Printed Name and Signature Martyn Kolloffel [signature]

For the week ending October 3rd 2003

☑ **By checking this box, I declare that I have devoted the equivalent of 20 hours of effort to this research project this week.**

Research Activities:

Library Research: - "Stochastic Population Models, compartmental perspective" by Matis & Kiffe
- "Imperfect maintenance" by Pham & Wang

Model Formulation: Discrete crack initiation, crack propagation rate is continues nonlinear function. Damage at time is analytical function (crack growth expression).

Key Research Findings:

- crack propagation can be studied as a deteriorating system. Availability function can be simulated using mathematical models. (Relate MGF to the intensity function!)

we need to establish "break-down" value..

Research Log
Investigations into the Application of Cumulant Functions in Operations Research and Stochastic Modelling

Printed Name and Signature MARTIJN KOLLOFFEL [signature]

For the week ending OCTOBER 10TH 2003

☑ By checking this box, I declare that I have devoted the equivalent of 20 hours of effort to this research project this week.

Research Activities:

Library Research: - Wang: "A survey of maintenance policies of deteriorating systems"
- Dieulle: "Reliability of several components sets at random time inspections"

Model formulation: Back to thingking about components. (not crack propagation)

Key Research Findings:

Formulation: A model in which "shocks" arrive to the components environment (according to Poisson). A Non Linear Intensity function relates the failure rate of the component to time t. Shocks cause non-fatal damage to the component. Component can be repaired, repair according to an exponential distribution.

Research Log
Investigations into the Application of Cumulant Functions in Operations Research and Stochastic Modelling

Printed Name and Signature MARTIJN KOLLOFFEL [signature]

For the week ending OKTOBER 17^{th} 2003

☑ By checking this box, I declare that I have devoted the equivalent of 20 hours of effort to this research project this week.

Research Activities:

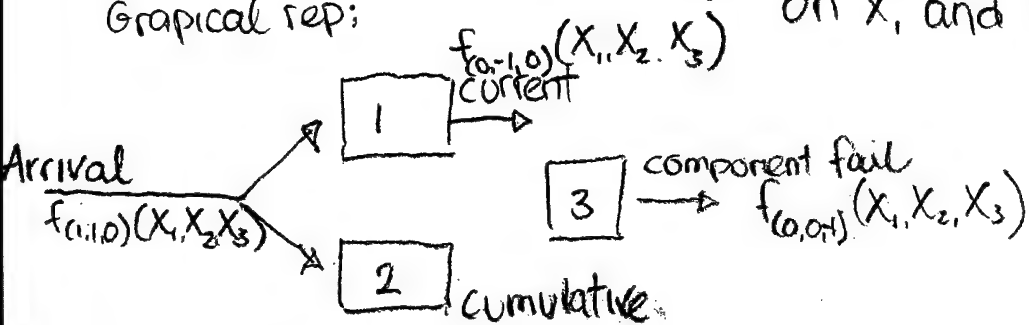
Library Research: - "On the value of an item subject to general repair or maintenance" by Scarcini & Shaked, M.
- "Multivariate Age-Dependent imperfect repair" by Shou, S, Griffith, W.S.

Model formulation - Impact or shock will cause repairable and cumulative Aging (or wear, damage). The rate of impact indicates the rate of aging of the specific environment. Service rate indicated by $f_{(0,-1,0)}(X_1X_2X_3)$ (intensity function) failing component $f_{(0,0,-1)}X_1X_2X_3$ dependent on X_1 and X_2

Key Research Findings:

Model formulation

Grapical rep:



$$E[X_3(t)] = \mu_3 X_3 + a X_3 X_1^2 + b X_3 X_2 X_1^2$$

a= effect of cumulative failures on rate
b= effect of current failures on rate

Research Log
Investigations into the Application of Cumulant Functions in Operations Research and Stochastic Modelling

Printed Name and Signature Martyn Kolloffel / M. Kolloffel

For the week ending OCTOBER 24th 2003

☑ By checking this box, I declare that I have devoted the equivalent of 20 hours of effort to this research project this week.

Research Activities:
Library Research: "Matis, T.I. Feldman, R.M. "Using Cumulant functions in Queuing theory"
Model Description: single component operating in an environment of non-fatal repairable shocks. Shocks may be system stresses or other components failing. Shocks arrive by Poisson process. Component is repaired with exponential server repair.
The failure of the component is Poisson with the rate being a function of both unrepaired of shocks received, and total number

Key Research Findings:
*Model Formulation:
Assume: initially component operational ⇒ $X_3(0)=1$
- No shock have arrived ⇒ $X_1(0)=X_2(0)=0$
- Intensity function corresponding to component failure $f_{(0,0,-1)}(X_1 X_2 X_3)$ is a non linear function of the current and cumulative number of shocks.
- (X_1) and (X_2) describe the damage process.
- Approximate the mean and variance of $X_1(t)$ & $X_2($
- Approximate the reliability function of the component: Expected value of $X_3(t)$
- Use cumulant derivation method to obtain approxim
- Compare to simulated values from Promodel.

Research Log
Investigations into the Application of Cumulant Functions in Operations Research and Stochastic Modelling

Printed Name and Signature MARTIJN KOLLOFFEL [signature]

For the week ending OKTOBER 31 2003

☒ By checking this box, I declare that I have devoted the equivalent of 20 hours of effort to this research project this week.

Research Activities:
Library Research: Kendall: "Advanced theory of Statistics"
Model Formulation: Use Mathematica program [updated version by Matis & Guardiola] to apply cumulant derivation procedure
- relate MGF to intensity function (PDE's)
- Convert MGF to CGF with Cumulant conversion.

Key Research Findings:
- Graph Expected values of $X_1(t)\, X_2(t),\, X_3(t)$
- Graph Variance of $X_1(t)\ X_2(t)\ X_3(t)$

* After some modifications in Mathematica, and a defenition of the intensity functions, Mathematica seems to be doing the correct calculations (compare to a "ballpark" range of expected value)

Research Log
Investigations into the Application of Cumulant Functions in Operations Research and Stochastic Modelling

Printed Name and Signature MARTIJN KOLLOFFEL [signature]

For the week ending NOVEMBER 7th 2003

☑ By checking this box, I declare that I have devoted the equivalent of 20 hours of effort to this research project this week.

Research Activities:

- Collect data on Mathematica Model
- Design a Simulation in Promodel that simulates the single component operating in the environment of non-fatal repairable shocks.

Key Research Findings:

* Comparison of Analytical and Simulated model will be done graphically. We calculate the variable of interest ($E(X_3(t))$, $VAR(X_3(t))$) at time t, and let t increase discretely from 0 to 50.
data is gathered for various truncation levels and varying defenition of intensity functions These data are entered into excel to allow graphical comparison to the simulation.

Research Log
Investigations into the Application of Cumulant Functions in Operations Research and Stochastic Modelling

Printed Name and Signature Martyn Kolloffel [signature]

For the week ending November 14, 2003

☑ By checking this box, I declare that I have devoted the equivalent of 20 hours of effort to this research project this week.

Research Activities:

* Design and Verify Promodel Simulation
* Library Research: "Simulation using promodel" by Harrel, Ghosh, Bowden

Key Research Findings:

* Nodes shown in graphical representation are insufficient to model this system. Extra nodes need to be adde
* Attributes are used to count current & cumulative
* Non linear function, relating rates and the states at the nodes, is calculated in processing logic
* Problems with real-time or continues updating of variabels. The precise moment at which the variabel changes in unclear.
  - Dr. Riley is asked to assist in solving this matter.

Research Log
Investigations into the Application of Cumulant Functions in Operations Research and Stochastic Modelling

Printed Name and Signature Martyn Kolloffel [signature]

For the week ending November 21st 2003

☑ **By checking this box, I declare that I have devoted the equivalent of 20 hours of effort to this research project this week.**

Research Activities:

- Redesign and verify Promodel
- Collect data from repetitions
- Compare Analytical and Simulation data
- Library Research: "Simulation using ProModel" by Harrel, Ghosh, Bowden

Key Research Findings:

- Problems regarding the exact timing of changing variables have been solved
  → WAIT UNTIL STATEMENT IN COMBINATION WITH A VARIABLE IS A SOLUTION
- Simulated model is verified and tested. It has been evaluated by outside sources and unanimous found to be a correct representation
- For a 5% confidence interval we need around 40.000 replications
- 10.000 repetitions are run first, to evaluate initial results.

Research Log
Investigations into the Application of Cumulant Functions in Operations Research and Stochastic Modelling

Printed Name and Signature MARTIJN KOLLOFFEL [signature]

For the week ending NOVEMBER 28, 2003

☒ By checking this box, I declare that I have devoted the equivalent of 20 hours of effort to this research project this week.

Research Activities:

- Graphically represent Simulation data as a reliability function (Surviver function) using excell.
- Verify Simulation
- Compare Analytical and Simulation data

Key Research Findings:

- The analytical data compare well to the Simulated data when graphically compared. using excel
- The relative diffence between approximation by Cumulative derivation method and Simulation, changes as the truncation level changes. (which could be expected and explained)
- Cumulative derivation method seems to be a good approximation method when evaluating the Survival function of a component that can be stochasically modelled.

Research Log
Investigations into the Application of Cumulant Functions in Operations Research and Stochastic Modelling

Printed Name and Signature MARTYN KOLLOFFEL M.Kolloffel

For the week ending DECEMBER 5th, 2003

☑ By checking this box, I declare that I have devoted the equivalent of 20 hours of effort to this research project this week.

Research Activities:
- Compare Simulation and Analytical data
- Draw Conclusions & write report.

Key Research Findings:
- Cumulant based Analysis of the Non fatal Shock process is very usefull when preventing system failure.
- This method can now be expanded to larger models to show that it is tractable for larger systems
- /

- Next week: GRADUATION !